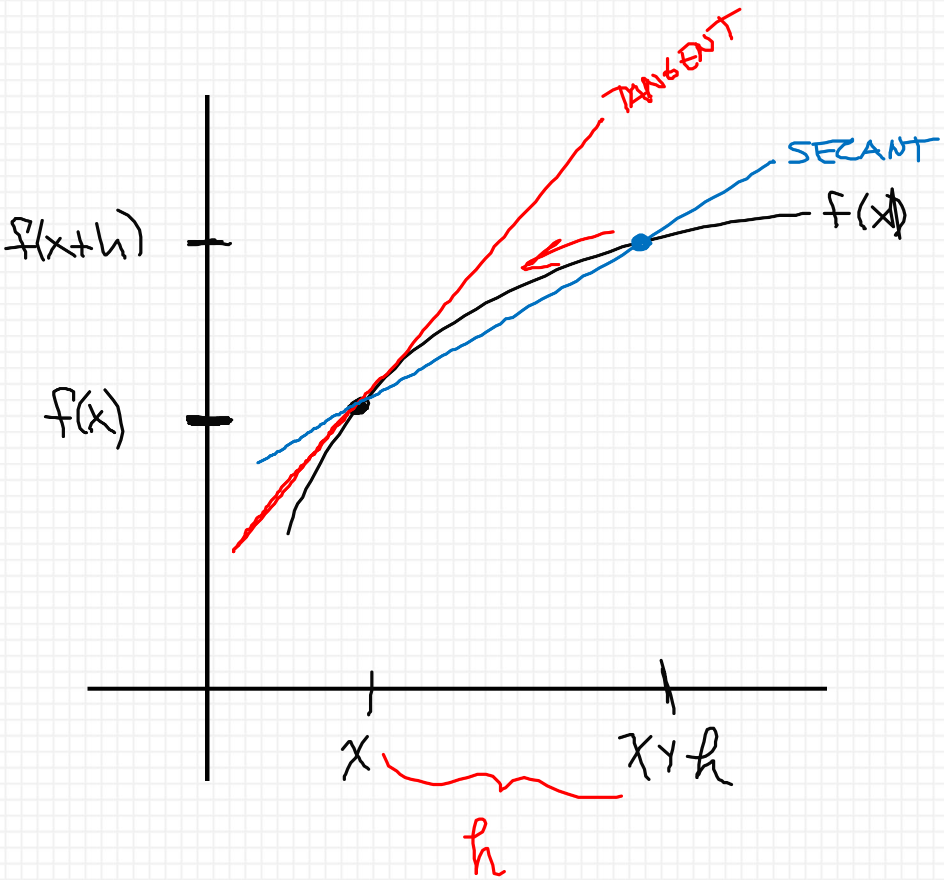
Chapter 3

Derivatives

3.1 - Definition of a Derivative



$$m_{SEC} = \frac{f(x+h) - f(x)}{x+h-x}$$

$$= \frac{f(x+h) - f(x)}{h}$$

$$\boxed{m_{TAN} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = f'(x)}$$

DEF. OF DERIVATIVE

Ex: Find f′ using the definition.

$$f(x)=\sqrt{x}$$

$$\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$$

"CONJUGATE"

$$\lim_{h \to 0} \frac{\sqrt{x+h}-\sqrt{x}}{h} \cdot \frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}$$

$$\lim_{h \to 0} \frac{\cancel{x}+\cancel{h}-\cancel{x}}{\cancel{h}\left(\sqrt{x+h}+\sqrt{x}\right)}$$

$$\lim_{h \to 0} \frac{1}{\sqrt{x+h}+\sqrt{x}} = \boxed{\frac{1}{2\sqrt{x}} = f'(x)}$$

Find the equation of the tangent line at x = 4.

$$m = f'(4) = \frac{1}{2\sqrt{4}} = \frac{1}{4}$$

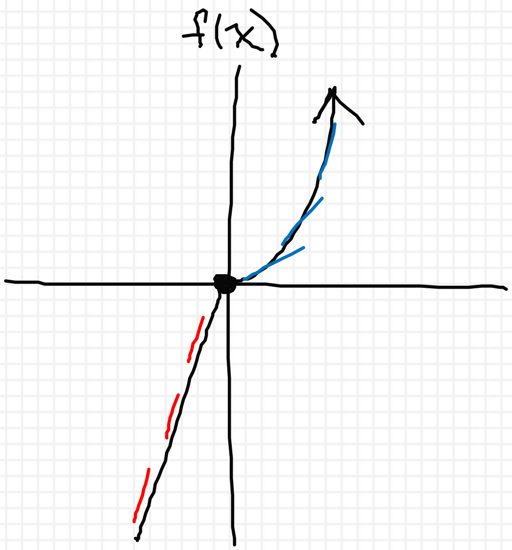
POINT : $f(4)=\sqrt{4}=2 \Rightarrow (4,2)$

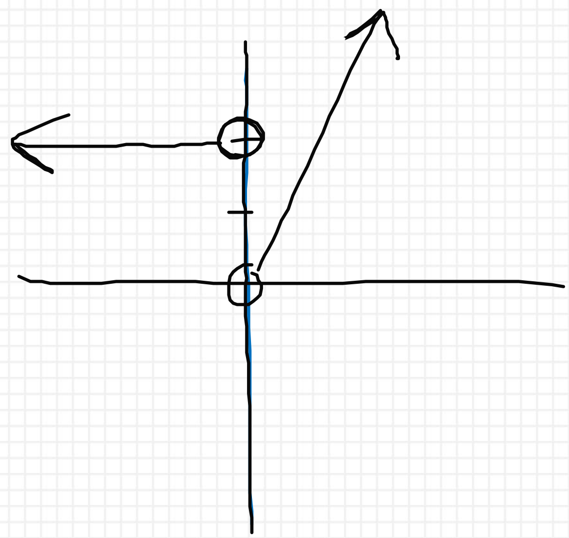
$$\boxed{y-2=\frac{1}{4}(x-4)}$$

Derivatives of Piecewise Functions

$f'(0)$ DNE

$$f(x)=\begin{cases}2x & x<0\\ x^2 & x\geq 0\end{cases} \Rightarrow f'(x)=\begin{cases}2 & x<0\\ 2x & x>0\end{cases}$$

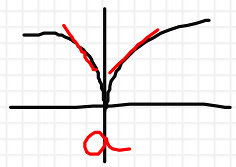




Section 3.2 – Differentiability

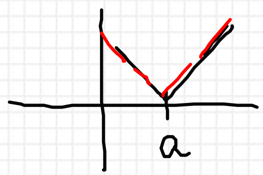
$f'(a)$ DOES NOT EXIST IF $x=a$ IS A(N)

① CUSP:



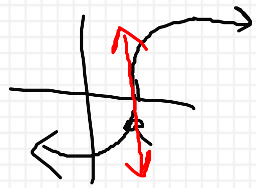
$\lim_{x\to a^+} f'(x) = \infty \Rightarrow$ DNE $\Rightarrow f'(a) =$ DNE

② CORNER:



$\lim_{x\to a^-} f'(x) \neq \lim_{x\to a^+} f'(x)$

③ VERTICAL TANGENT:



$\lim_{x\to a} f'(x) =$ DNE (VERT LINE)

④ DISCONTINUITY @ $x=a$

Section 3.2 – Differentiability

Theorem: If $f(x)$ is differentiable at a point a then $f(x)$ is continuous at $x = a$. DIFFERENTIABILITY IMPLIES CONTINUITY

Converse: IF $f(x)$ IS CONTINUOUS AT $x=a \Rightarrow f(x)$ IS DIFF. @ $x=a$ ..

☆ NOT TRUE ☆

Section 3.3 – Differentiation Rules (handout)

Section 3.5 – Trig Derivatives

Section 3.6 – Chain Rule

$g(t) = \tan(5 - \sin 2t)$

$g'(t) = \sec^2(5 - \sin 2t)\left(-\cos(2t)\, 2\right)$

$= -2\cos(2t)\sec^2(5 - \sin 2t)$

$f(x) = (\cos 3x)^5$

$f'(x) = 5(\cos 3x)^4(-\sin 3x \cdot 3)$

$= -15 \sin 3x \cos^4 3x$

Section 3.8 – Inverse Trig Derivatives

$$\frac{d}{dx}\tan^{-1}x = \frac{1}{1+x^2}$$

$f(x) = \sin^{-1}(3x)$

$$\frac{d}{dx}\sin^{-1}x = \frac{1}{\sqrt{1-x^2}}$$

$$f'(x) = \frac{1}{\sqrt{1-(3x)^2}} \cdot 3$$

$$= \frac{3}{\sqrt{1-9x^2}}$$

$y = x\tan^{-1}\left(x^2\right)$

$$y' = x \cdot \frac{1}{1+(x^2)^2} \cdot 2x + \tan^{-1}(x^2)\cdot 1$$

$$= \frac{2x^2}{1+x^4} + \tan^{-1}(x^2)$$

Section 3.9 – Logs and Exponentials

$$\frac{d}{dx}\left(b^u\right) = b^u \ln b \cdot \frac{du}{dx}$$

$$\frac{d}{dx}\left(\log_b u\right) = \frac{1}{u \cdot \ln b} \quad \frac{du}{dx}$$

Homework

AP Packet #24 – 48 odd